Pedestrian-Inspired Sampling-Based Multi-Robot Collision Avoidance

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Abstract-We present a distributed collision avoidance algorithm for multiple mobile robots that is model-predictive, sampling-based, and intuitive for operation around humans. Unlike purely reactive approaches, the proposed algorithm incorporates arbitrary trajectories as generated by a motion planner running on each navigating robot as well as predicted human trajectories. Our approach, inspired by human navigation in crowded pedestrian environments, draws from the sociology literature on pedestrian interaction. We propose a simple two-phase algorithm in which agents initially cooperate to avoid each other and then initiate civil inattention, thus lessening reactivity and committing to a trajectory. This process entails a pedestrian bargain in which all agents act competently to avoid each other and, once resolution is achieved, to avoid interfering with others' planned trajectories. This approach, being human-inspired, fluidly permits navigational interaction between humans and robots. We report experimental results for the algorithm running on real robots with and without human presence and in simulation.

I. INTRODUCTION

Sampling-based planners have often been employed to enable robots to avoid obstacles, both static and dynamic [2]. If the obstacle is an intelligent agent, such as a human or another robot, this problem is complicated by the difficulty in predicting the agent's reaction to the robot's own movements (Fig. 1). A number of multi-robot collision avoidance algorithms have been proposed that can incorporate the reactions of fellow robots. However, many of these, such as potential fields [27] and reciprocal velocity obstacles [28], are purely reactive and therefore cannot incorporate general predictions about an agent's intended trajectory.

We describe the first distributed, sampling-based, cooperative collision-avoidance algorithm that is predictive, reactive, and reciprocal, thus leaving the system of robots free from instability. We have demonstrated a mixed human-robot team performing interactive collision avoidance smoothly and safely. The key insight—stemming from observations of humans navigating in pedestrian traffic—is that people normally perform pairwise mutual avoidance by reacting to each interfering person only once. This simple behavior, which enables humans to smoothly avoid one another, can be easily recreated within the sampling-based planning paradigm.

A. Prior Work

From a motion planning perspective, intelligent, dynamic obstacles open up many avenues for investigation. Many have noted that multi-agent collision avoidance is a cooperative



Fig. 1. Four robots cooperate with each other and a human at an assembly task [24] in a confined space. The robots' navigation and collision avoidance algorithms draw inspiration from human social conventions for moving in crowded pedestrian spaces.

pursuit, leading to reciprocal algorithms in which an agent can expect its co-agents to cooperate in avoiding each other [13, 23, 26]. Trautman and Krause [26] recognize that if agent trajectory prediction is done for the purpose of robot navigation, then it is vital to perform joint collision avoidance by incorporating the robot's own planned motions into the prediction to avoid deadlock.

The reciprocal velocity obstacle concept and its variants [23] represent a reactive approach to collision avoidance in which moving objects are assumed to possess a desired constant velocity. Under a naive assumption that other robots do not react, an oscillation termed reciprocal dance can arise as robots repeatedly react to changes in each others' actions. To solve the resulting oscillatory behavior, the reciprocal velocity obstacle (RVO) approach assumes that all other agents are also running the RVO algorithm and will cooperatively react to avoid the collision. Each robot then commands the constant velocity that deviates minimally from its preferred trajectory. The hybrid reciprocal velocity obstacle is an enhancement on the RVO concept, in which moving to the right is preferred by allowing 50% avoidance to the right but requiring 100% avoidance to the left.

Wilkie et al. [29] propose a sampling-based implementa-

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tion of the velocity obstacle concept, which assumes constant velocity on the part of all moving obstacles. They sample in the space of controls, as we do, and they apply the velocity obstacle concept to nonholonomically constrained mobile robots. Reciprocal behavior is not addressed.

Helbing and Molnár [8] offer an alternative type of reactive model for pedestrians moving in crowds. Like the potential fields [11] of robotics, their social fields incorporate repulsive forces to avoid other pedestrians and an attractive force to pull an agent toward its goal. This model can also incorporate penalties for walking in non-preferred areas. The work of Treuille et al. [27] employs such social field concepts to simulate thousands of believable pedestrians in real time.

These social field methods are inherently "zeroth-order" models of pedestrian behavior in the sense that an individual's velocity is governed by only the position of neighboring agents—as though their motions are a surprise. The reciprocal velocity obstacle methods, which anticipate a collision avoidance reaction, are still only first-order reactive methods in that they otherwise expect indefinite constant velocity among all agents.

Kirby [12] argues that "appropriate social behavior requires optimal global planning for obstacle avoidance, rather than locally reactive behaviors." We introduce such a global planning approach to collision avoidance, but a commensurate pedestrian prediction algorithm is required to empower robots to operate amongst humans.

The prediction of pedestrian trajectories has been well studied. Even the problem of predicting a single pedestrian in isolation can lead to sophisticated solutions [16, 25, 31]. Another approach has been to model pedestrian behavior in aggregate within crowds [4, 9].

In planning robot motions near people, there has been work in incorporating human social comfort into robot motions [12, 21, 22]. These planners understand that pedestrians move, and they incorporate human factors like proxemics, visibility, and passing side. However, none of these humanaware planning approaches is capable of utilizing more than a first-order model of pedestrian prediction.

This paper contrasts with prior work by incorporating the human practice of *civil inattention* to make robot motions more socially acceptable. Furthermore, it employs a sampling-based motion planning algorithm that is simultaneously predictive and reactive and thus is compatible with human environments. By employing a reciprocal planner, we ensure consistent, intuitive behavior so long as other agents' motions can be predicted.

II. DYNAMIC OBSTACLE AVOIDANCE ALGORITHM

Dynamic obstacle avoidance is contingent on two separate capabilities. First, the robot must be able to predict the future trajectory of a dynamic obstacle, or agent, passing through the robot's environment. For other robots running the algorithm, prediction is performed by by exchange of timeparametrized planned trajectories. When interacting with other robots or humans, it is necessary to run a prediction algorithm. Second, the robot must define a control strategy



Fig. 2. Cooperative collision avoidance from the left agent's perspective. **Top:** The agent detects another agent on a colliding trajectory. **Middle:** The agent plans a maneuver that avoids at least 50% of the impending overlap. **Bottom:** The pedestrian bargain requires that the oncoming agent plan a similar maneuver, thus separating the agents' centers by at least 100%.

that is both optimized for the predicted trajectory and safe in any other outcome. We begin this section with a brief overview of the sociology literature concerning pedestrian avoidance models.

A. A Primer on Human Avoidance Models

In navigating through personal spaces, humans make frequent, minor corrections to their trajectory in response to the predicted motions of other people. In so doing, we follow a social convention, or pedestrian bargain, designed to distribute responsibility for altering one's trajectory in recognition of another's intentions. Wolfinger [30] describes the pedestrian bargain as comprising two rules: "(1) people must behave like competent pedestrians, and (2) people must trust copresent others to behave like competent pedestrians."

In describing *unfocused interaction* [7, P. 24], Goffman illustrates one key aspect, *civil inattention*: following acknowledgment of a person, one looks away "so as to express that he does not constitute a target of special curiosity or design" [7, P. 84].

The following procedure, based on observation of pedestrians, encapsulates the above concepts. We portray the procedure from the point of view of a person (the self), who is interacting with a single pedestrian (the other).

Pedestrian Avoidance Procedure:

- 1) The interaction begins when the self perceives a possible future collision with the other.
- 2) *If the other appears competent and engaged*, then the self makes a visible move to correct their trajectory by about half of the amount required to fully avoid collision with the other.
- 3) Otherwise, the self makes a full effort to avoid the other.
- 4) Finally, the self resolves the interaction by initiating civil inattention.

Most often, this sequence occurs simultaneously on the part of both pedestrians engaged in an interaction, who thus both fulfill their half of the bargain (Fig. 2). However, the procedure is robust to various circumstances. Suppose one pedestrian is distracted (i.e. practicing civil inattention). The alert person, A, must estimate whether he believes

the distracted one, D will eventually fulfill his half of the bargain. If A chooses to fully avoid D, then the interaction is resolved (at the cost of A thinking D rude). If, on the other hand, A believes that D will come around in time to interact, then A may perform the half-correction. By the time D does notice the incipient collision, A has begun to exercisers civil inattention, thus failing the *engaged* qualification. Consequently, D must make 100% of the effort to avoid collision—that amount being the original 50% effort, had D been paying attention.

B. Dynamic Agent Control Strategy

The decentralized control strategy for an agent navigating among other moving agents can be derived directly from Table I. The policy prescriptions in the right column dictate the amount of the total initiative that agent A should assume on the basis of the assumption stated in the left column. The ReactAlone algorithm assumes that the other agent will make no reaction. In contrast, ReactCooperatively behaves as though the other agent will also react (as dictated by the pedestrian bargain). Concretely, a cooperative reaction means that two agents share responsibility for avoiding each other—they should each make half of the total effort.

TABLE I DISTRIBUTED COOPERATIVE REACTION POLICY

If agent $B \dots$,	then agent A should
did react	ReactAlone
will react	ReactCooperatively
will not react	ReactAlone

The greatest challenge in implementing this policy comes in classifying the intention of other agents. For the case of robot-robot interaction, the robots can simply communicate their intended trajectory and with what other agents they are currently interacting.

In the case of a human and robot interacting, the robot must track and predict the motion of the pedestrian. Sophisticated pedestrian predictions that represent arbitrary trajectories, such as the work of Ziebart et al. [31], are supported. At present, we employ a simpler constant-velocity prediction algorithm as in the work of Kirby [12]. At present, we do not attempt to communicate civil inattention on the part of robots towards people.

We assume some maximum speed v_{max} and lookahead time τ for all agents, which is sufficient to guarantee that all agents can react to one another and prevent collision. Thus, inevitable collision states [6] can be avoided. Together, these parameters give rise to a neighborhood comprising all space within the controller horizon distance of $d_{max} = v_{max}\tau$. The controller calls for each robot to broadcast to others within radius d_{max} a list of IDs of robots to which it has already reacted. Robots also broadcast their own latest planned trajectory.

III. SAMPLING-BASED IMPLEMENTATION

In this section, we describe a sampling-based motion planning implementation for multi-robot collision avoidance.



Fig. 3. Depiction of the sampling-based planner in action. The red robot at top left is moving towards its goal at bottom right while avoiding the other agents' predicted trajectories in purple. Each agent's local planner samples 120 controls representing possible commanded velocities. Paths surviving the collision test are classified (colored) according to routes among obstacles, similar to a localized version of homotopy (see [15]). The planner selects one path for execution (red with dot). Note that the robot plans spatio-temporally, thus the red path passes behind the purple robot. The result of four robots executing the algorithm is an emergent traffic circle pattern.

Sampling-based motion planners operate on the principle of rejection sampling. Time-parametrized paths are drawn from some distribution that neglects any obstacles in the environment. The sampled paths are then collision checked, and any path that would result in a collision is rejected from further consideration by the planner. This approach may be costly in densely cluttered spaces, but it provides two helpful attributes: (1) sampling bias is minimized because the distribution of paths in the free space is precisely the underlying sampling distribution within that space, and (2) the rejection sampling algorithm is simple to implement.

A. Model-Predictive Hierarchical Planner

We build on our implementation of the model-based hierarchical planner [14]. Hierarchical planners have a long history in navigation [1, 5, 17, 19]. Many sampling-based planners are amenable to the ideas we present, but the hierarchical planner is particularly suited to local pedestrian avoidance. Its path samples are limited to the local neighborhood around the robot—the same space with which humans are concerned while navigating through pedestrian traffic.

In the planner hierarchy, planners are arranged in levels that trade off predictive fidelity against range. This architecture is motivated by the notion that the majority of computational cycles should be spent on the most immediate future. Kinematics and dynamics are the most constraining over short intervals of space and time, thus making planning the most challenging at this range. Also, the robot is most certain about the position and trajectory of other agents in the space closest to its sensors.

At the local level, the planner is fully aware of the robot's kinodynamic constraints based on a predictive model. It

ensures feasibility of local trajectories by sampling controls from the action set and integrating them forward in time.

Figure 3 shows the candidate paths in several colors to indicate a classification of the paths according to the predicted motion the robot would make with respect to obstacles. This equivalence relation [15] allows the planner to cluster similar paths that permit a continuous deformation in path shape, resembling homotopy. However, these path deformations obey all robot motion constraints and are restricted to a limited area around the robot's current state.

This equivalence relation is valuable for collision avoidance. Given a nominal path, the planner is able to substitute a variety of alternative paths with the knowledge that each trajectory would pass each other agent on the same side as the original, thus not provoking a major change in others' plans. In particular, we use this capability to select among a variety of safe paths that are equivalent to the shortest path to the goal. The shortest path tends to graze obstacles, which is undesirable both for safety and social acceptability. The robot therefore prefers to execute a path that is in the same equivalence class as the shortest path but is appropriately far from the closest obstacle (whether static or dynamic). An additional benefit of this approach is that the shared-initiative collision avoidance algorithm tends to converge quickly.

At the global level, a long-range grid-based motion planner abstracts away kinodynamic constraints but retains awareness of the topology of the free space. The global planner thus provides a value function that guides the robot in the direction of the goal.

In order to mediate between the local and global planners, the local planner possesses a time horizon similar to that of model-predictive control. Beyond this horizon, the global planner serves as a heuristic to estimate time to the goal. We assume that the local planner's horizon, τ , is sufficient to safely avoid other moving agents. This assumption, coupled with robot dynamics, constrains the speed at which robots may safely move through space.

In order to react to changing conditions and maintain a safe reaction distance as the robot moves through the world, the local planner must replan at a regular rate $r \gg \frac{1}{\tau}$. For pedestrian-scale dynamics, a replan rate on the order of 10 Hz is typically sufficient. At the conclusion of each replan cycle, the planner must select a single path to execute from among those that survived a collision check. The path selected is that which minimizes the sum of local and global time to the goal.

B. Sampling-Based Cooperative Collision Avoidance

As with conventional rejection sampling, our planner collision checks a sequence of paths and rejects any sample predicted to result in a collision with a static obstacle in the environment. This planner additionally models and avoids the trajectories of dynamic obstacles (agents) in the environment. Agent models include human predictions as well as planned trajectories communicated from other robots.

Unlike static obstacle collisions, we do not necessarily reject a path sample that is predicted to collide with a moving agent's current trajectory—after all, that agent should be expected to cooperate in the avoidance maneuver.

There are two phases of interaction as indicated by the presence or absence of civil inattention. Before the onset of civil inattention, we compute the maximal expected overlap between the trajectories of the robot and oncoming agent as a fraction of agent diameter. Overlap fraction is used as a proxy for the expected contribution of the other agent to collision avoidance. Trajectories that overlap by more than some fraction are rejected by the planner. The rejection of samples with overlap above 50% would result in balanced initiative between the robot and agent.

In practice though, the agent may be unable to avoid the robot by 50%, perhaps because of another obstacle. The planner therefore follows a progression, in which the threshold for rejection reduces overlap with each successive iteration. Let the overlap rejection threshold at replan iteration i be denoted f_i . With the first iteration since initiating an interaction, let $f_0 = \frac{1}{2}$. Subsequent thresholds are defined by a progression $f_{i+1} = \frac{f_i}{r}$, with r > 1. The rate of the progression, r, can be tuned as appropriate for a given response time based on the replan rate. This progression enables the robot to rapidly adapt to the agent's intentions, while still behaving socially with agents that follow the protocol. Note that at each step, the use of path equivalence classes often leads the robot to prefer paths with less overlap, as they are also safer to execute. Thus, the process generally converges quickly.

After one agent initiates civil inattention with respect to another, the second agent must only execute trajectories that entirely avoid the first. That is, all future $f_i = 0$. Once it has found a collision-free path, the second agent would naturally also initiate civil inattention with respect to the first. Being unable to detect civil inattention in humans, we currently assume humans always engage in civil inattention towards all robots, causing robots to defer to humans.

Finally, it should be noted that two robots that have already reacted to one another and initiated civil inattention may need to reinitiate an interaction at a later time. One such scenario occurs when the robots depart and reconverge. In such a case, they could view each other as novel agents.

A separate case for turning off civil inattention occurs when something unanticipated changes in the environment. For instance, the action of some third agent may eliminate a robot's prior plan, leaving it only paths that would collide with some other agent. In such a case, the robot would be forced to re-engage with all its neighbors.

C. Passing Convention

The above algorithm results in both agents nudging their trajectories sufficiently sideways in order to avoid collision. However, it is important that the agents do not move to the same side. Thus, a convention is needed for passing. In evaluating two agents' trajectories, we examine their points of closest approach. Agent A considers the side of agent B on which it passes. We follow the North American convention that A passes on B's left, as shown in Fig. 4.



Fig. 4. A passing convention is needed that covers diverse situations. We adopt the convention that agent A prefers to pass to co-agent B's left side. This rule fits expected behavior for both oncoming passes (left) and overtaking passes (right) in North America.

We implement the side preference using overlap fractions. A sampled path that passes on the preferred side is subject to the aforementioned threshold of $f_i \leq \frac{1}{2}$, whereas trajectories passing on the nonpreferred side are subject to an elevated threshold of $e_i = f_i - 1$. An overlap of $-\frac{1}{2}$ indicates that a gap of one robot radius must exist between the two agents. This asymmetry causes the robot to prefer to pass to another agent's left because it results in a smaller, less costly deviation from the desired trajectory.

D. Contingency Plans

As a parallel distributed system, occasions will arise in which the robots must iterate several times before resolving an interaction—just as pedestrians sometimes reach a stalemate in which they both attempt to move to the same side several times. In such instances, the stochasticity introduced by the sampling-based planning algorithm is a valuable asset in breaking the symmetry and resolving the stalemate.

To guarantee safety, however, it is insufficient to rely on stochasticity. In addition to computing the primary preferred trajectory, we ask the planner to supply a second, contingency trajectory with two properties. First, the contingency trajectory is free of collision under the more conservative assumption that the opposing agent does not cooperatively alter its trajectory. Second, the contingency trajectory shares its initial control input with the primary trajectory, then deviates to avoid collision. The contingency is constructed so that if no viable trajectories are found in the subsequent replan cycle, then the contingency trajectory may still be safely executed. Bekris et al. [3] describe a similar contingencybased distributed motion planning algorithm with safety guarantees. For mobile robots, the contingency trajectory generally correlates to a stop. Any contingency trajectory has the property that it can be executed in perpetuity. Thus, the planners on each robot have sufficient time to work out any situation to avoid deadlock.

E. Guarantees

By virtue of utilizing a complete motion planner, the collision avoidance algorithm provides several important guarantees when robots avoid other robots. These guarantees include being deadlock-free, livelock-free, and starvation-free. A brief argument follows.

Completeness is the property that a motion planner will find a solution in finite time to every query for which a solution exists, and it will otherwise report that no solution exists. MBHP provides a limited form known as resolution completeness, which provides that if a solution exists within a given sampling resolution, it will be returned in finite time.

By exchanging planned trajectories, the individual robots have sufficient information to guarantee resolution completeness for the collection of motion planners in a local neighborhood. The completeness property provides that when convergence occurs, the collection of planners has found a solution to the collective motion planning problem that includes trajectories for all interacting robots.

The condition of possessing a solution to the collective motion planning problem naturally leads to each of the guarantees. Unlike pure collision avoidance schemes, a motion plan leading all the way to each robot's goal ensures that no robot will become stuck in an infinite wait (deadlock, starvation) or execute a series of motions that fail to make progress to the goal (livelock).

IV. EXPERIMENTAL RESULTS

We performed experiments on the collision avoidance algorithm both in simulation and with real robots. We employed ROS [20] for communication among the robots. For the purpose of these experiments, the robots were provided accurate localization; incorporation of localization uncertainty into the algorithm remains a subject for future work. We localized the real robots using a Vicon tracking system equipped with twelve cameras. Simulated physics and localization were provided by Gazebo as packaged with ROS. In simulation, we studied scalability and the algorithm's ability to handle crowded circumstances. In the real robot experiments, we examined human interaction while the robots performed an assembly task.

A. Real Robot Results

With the real robots, we demonstrated the ability of the collision avoidance algorithm to successfully complete a task involving teamwork amongst robots while safely accommodating a human moving through the environment. The assembly task (Fig. 1) involves a team of four KUKA youBots delivering and assembling components to build a tower, based on the algorithm of Stein et al. [24]. During the trials, an expert user wandered through the 5 m x 5 m robot workspace, both aiding and interfering with the robots.

For these experiments, the human wore a hat marked with retroreflective tracking markers tracked by the Vicon system. A sliding window average of one second was used to estimate the human's current velocity. The trajectory was then predicted by integrating ten seconds into the future.

We ran five trials of the assembly procedure over a combined seventeen minutes. During that time, we recorded 67 instances in which a robot reacted to the human, averaging one such interaction every fifteen seconds. Eighteen interactions involved the human standing at the robot's goal or blocking the robot's path. In such circumstances, the robot has no means of progressing, and so it sits and waits for the human to move. These behaviors are identical to the way



Fig. 5. We performed experiments in which robots are placed in a circle of 8 m diameter and asked to swap positions with their antipodal neighbor. The bottom figure shows an intermediate state in a sixteen-robot navigation scenario. Although the scene appears chaotic, a pattern of counterclockwise traffic circle flow emerges.

that robots treat other robots. In over sixty human attempts to block the robot, only a single contact occurred, in which the robot brushed the human's foot. This occurrence is believed to be a result of lag in the prediction algorithm reporting to the planner since the tracking algorithm runs off-board.

B. Simulation Results

We ran simulation trials in order to demonstrate statistically significant results over larger numbers of trials and also to demonstrate scalability of the algorithm beyond the number of physical robots available to test. We performed experimental runs on groups of robots ranging from one to sixteen. In these results, we arranged the robots in an 8 m diameter circle and commanded each to swap positions with its antipodal neighbor.

A traffic circle pattern naturally emerges, although the algorithm was not explicitly designed to do so. This result suggests that the algorithm correctly captures the passing semantics adopted by humans. Figure 5 shows an example of the crowded conditions under which the algorithm ran.

Figure 6 demonstrates the scalability of the algorithm. As the density and quantity of robots attempting to navigate through the crowd increases, the average time for each to reach its destination scales linearly, suggesting that our decentralized approach remains efficient and effective even in dense crowds. Reported times represent the total planning, execution, and wait time. Note that since the each robot only interfaces with others in its fixed-area neighborhood,



Fig. 6. Simulated scalability experiment. Robots are placed in a fixed diameter circle as in Fig. 5. As the number of robots increases, the impact on elapsed time (sum of planning, execution, and wait time) for all robots to reach their goal is shown. Error bars indicate range of observed times over sixteen trials. Average time for a robot to reach its destination appears to scale linearly with the number of robots. No collisions or deadlocks occurred during any experiment.



Fig. 7. In this experiment, each solid robot must exchange positions with its diagonally-opposite neighbor while avoiding collision with walls and other robots. The robots all simultaneously reach the narrow doorway, which permits only one robot at a time to pass through. The resulting behavior is safe and efficient, with both robots on one side holding back (solid line pausing at ghosted robots) while the other two (dotted line) pass through in tandem. Once the second robot clears the doorway, the two waiting robots proceed through.

the performance relates only to the local population within that neighborhood—that is, the density of robots.

Finally, in Fig. 7, four robots must navigate through an office-like environment. A narrow doorway at the center permits only one robot to pass through at a time, thus forcing them to take turns. As in the case of boarding a train, it is most efficient for all agents to pass through in one direction while the others wait for their turn, and this is precisely the behavior we observe.

V. DISCUSSION AND FUTURE WORK

In this paper, we propose a new multi-agent distributed collision avoidance algorithm. This algorithm avoids the drawbacks of purely reactive collision avoidance algorithms by anticipating entire trajectories for each agent involved in the interaction. Trajectories are selected using a dynamicsaware sampling-based motion planning algorithm. The avoidance of moving agents—both robots and humans—can naturally be integrated into this motion planning framework. Any dynamic system that can be modeled is supported. The algorithm draws inspiration from human pedestrians to cooperatively avoid collisions without prioritizing one agent over another. By introducing both prediction and civil inattention into the process, we prevent chain-reactions among agents, in which each reacts to another's reaction.

We have demonstrated the algorithm in a mixed humanrobot setting. Future extensions of this work include the incorporation of more social conventions, including the use of gaze (both to and from the robots), which will result in improved performance for the collision avoidance algorithm.

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